Large Signal Analysis of Optical Directional Coupler Modulators

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Abstract—**This paper deals with the large-signal analysis of optical directional coupler modulators. Expressions are obtained for the output harmonic and intermodulation products of such devices when driven by multisinusoidal input signals. The special case of relatively small input amplitudes is considered. The results presented are verified through comparisons with previously published results.**

I. Introduction

In a recent publication Halemane and Korotky[1] present a theoretical analysis of the nonlinearities of the intensity modulation response of the interferometric, the directional coupler and the y-fed coupler modulators. According to Halemane *et al.* the nonlinear characteristic of the modulator can be expressed by (1):

$$\frac{I}{I_o} = f\left(\frac{V}{V_s}\right) \tag{1}$$

where

I	is the optical intensity output of the modulator
I_o	is the input optical intensity
$V = V_b + V_m$	is the applied voltage
V_b	is the bias voltage
V_m	is the modulating voltage
V_s	is the minimum voltage swing required to produce 100% modulation depth of the optical intensity.

$$f\left(\frac{V}{V_s}\right) = \frac{1}{2}\left(1 + \cos \pi \frac{V}{V_s}\right) \tag{2}$$

for the interferometric modulator,

$$f\left(\frac{V}{V_s}\right) = \frac{\sin^2\left(kL\sqrt{1 + 3\left(\frac{V}{V_s}\right)^2}\right)}{1 + 3\left(\frac{V}{V_s}\right)^2}, \qquad -1 \le \frac{V}{V_s} \le 1 \tag{3}$$

for the directional coupler modulator, where k is the optical coupling coefficient and L is the device length, and

$$f\left(\frac{V}{V_s}\right) = a^2 \cos^2\left(\frac{\pi r}{2}\right) + [(ax - by)/r]^2 \sin^2\left(\frac{\pi r}{2}\right) \tag{4}$$

for the y-fed coupler modulator, where $X = \Delta\beta L/\pi$, $y = L/l$, $r^2 = X^2 + y^2$, L is the length of the electrodes, l is the coupling length, $\Delta\beta$ is the induced phase mismatch, and a^2 and b^2 are the power fractions in the top and bottom arms at the input of the coupler. For the directional coupler modulator, Halemane *et al.* specialised their interest in the usual case of $kL = \pi/2$ which corresponds to the device shortest length for a given k. Thus, (3) reduces to

$$f\left(\frac{V}{V_s}\right) = \frac{\sin^2\left(\frac{\pi}{2}\sqrt{1 + 3\left(\frac{V}{V_s}\right)^2}\right)}{1 + 3\left(\frac{V}{V_s}\right)^2}. \tag{5}$$

Also, for the y-fed coupler modulator, Halemane et al. specialized their interest in the case of symmetric branching ($a^2 = b^2 = 0.5$). With $y = (1/\sqrt{2})$, which corresponds to maximum modulation, and by normalizing the voltage so that $X = \sqrt{2}\,(V/V_s)$, (4) reduces to

$$f\left(\frac{V}{V_s}\right) = \frac{1}{2}\cos^2\left(\frac{\pi}{2}\sqrt{\frac{1}{2} + 2\left(\frac{V}{V_s}\right)^2}\right) + \left[\frac{\left(\frac{V}{V_s} - \frac{1}{2}\right)^2}{\frac{1}{2} + 2\left(\frac{V}{V_s}\right)^2}\right]\sin^2\left(\frac{\pi}{2}\sqrt{\frac{1}{2} + 2\left(\frac{V}{V_s}\right)^2}\right),$$
$$-\frac{1}{2} \le \frac{V}{V_s} \le \frac{1}{2} \tag{6}$$

Figs. 1–3 show plots of (2), (5) and (6) respectively, from which it is obvious that these equations are nonlinear. Therefore, if the applied voltage V is formed of a multisinusoidal signal, harmonic and intermodulation products will appear in the optical intensity output of the modulator. However, while (2), in its present form, can be used for predicting the amplitudes of the harmonics and intermodulation products of the optical intensity output of the interferometer modulator [2], (5) and (6) can not be used in the present forms. Therefore, Halemane *et al.* specialized their interest in the small signal conditions, under which (1) can be expanded in a Taylor series about V_b. By truncating this Taylor series after the fourth term and assuming that the modulating voltage is formed of two tones with relatively small amplitudes, Halemane *et al.* obtained expressions for the third-order intermodulation, P_{3IM}, second-order intermodulation, P_{2IM}, and the third-order intercept point (defined as the operating point where P_{3IM} equals the carrier power P_c when extrapolated from the small-signal regime). The analysis of Halemane *et al.* cannot, therefore, predict the harmonic and intermodulation performance under large signal conditions. It is the purpose of this paper to present simple approximations for (5) and (6). These approximations, which are valid over the full useful range of input voltage, are intended to provide simple analytical expressions for the harmonic and intermodulation performance of the interferometric, directional coupler and the y-fed coupler modulators under large signal conditions. Such analytical expressions are important not only for evaluating the performance of these modulators when used in analog communication systems to send multiple channels simultaneously but also to evaluate their use as harmonic mixers, multipliers [2] and electromagnetic field detectors [3].

II. Proposed Model

The development of this model has proceeded along empirical lines by comparing the truncated Fourier-series model of (7) with the characteristic of (5) for the directional-coupler modulator:

$$f\left(\frac{V}{V_s}\right) = A_0 + \sum_{n=1}^{6} A_n \cos\left(n\pi \frac{V}{V_s}\right), \qquad -1 \le \frac{V}{V_s} \le 1 \tag{7}$$

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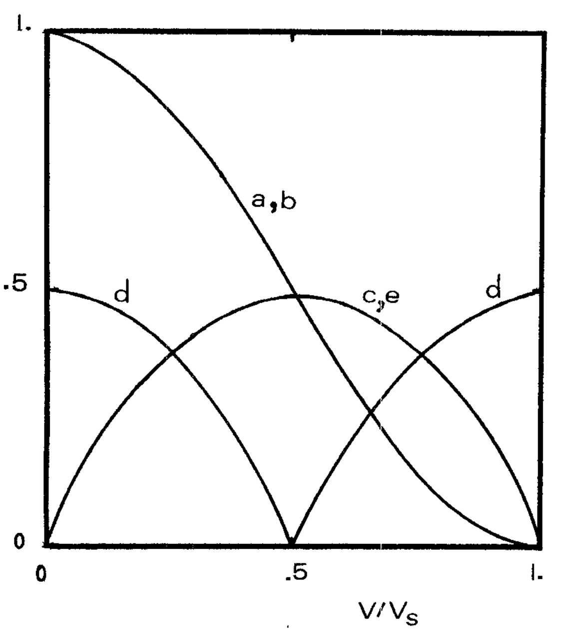

Fig. 1. The interferometer modulator characteristic $f(V/V_s)$ of (2) (marked a) and the corresponding functions F_0, F_1, $|F_2|$, and F_3 of (27)–(30) (marked as b, c, d and e).

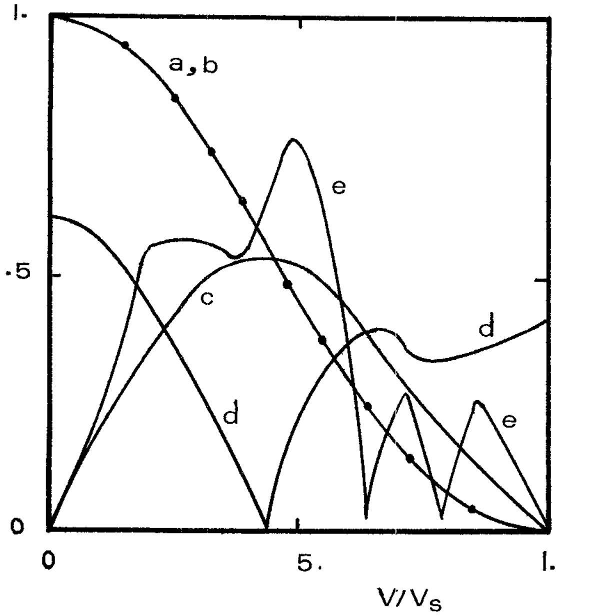

Fig. 2. The directional coupler modulator characteristic $f(V/V_s)$ of (5) and (7) (marked a) and the corresponding functions F_0, F_1, $|F_2|$ and F_3 of (27)–(30) (marked as b, c, d and e). —— (5); • (7).

Also, the truncated Fourier-series model of (8) is compared with the characteristic of (6) for the y-fed coupler modulator.

$$f\left(\frac{V}{V_s}\right) = A_0 + \sum_{n=1}^{6} A_n \cos\left(n\pi\left(\frac{V}{V_s} + 0.5\right)\right),$$
$$-\frac{1}{2} \leq \frac{V}{V_s} \leq \frac{1}{2} \tag{8}$$

The parameters A_n, $n = 0$–6 of (7) and (8) can be obtained using the twelve-point method (4) and are shown in Table I. Also shown in Table I are the parameters A_n, $n = 0$–6 of (2).

To establish the accuracy of (7) and (8), calculations were made and are shown in Figs. 2 and 3, from which it can easily be seen

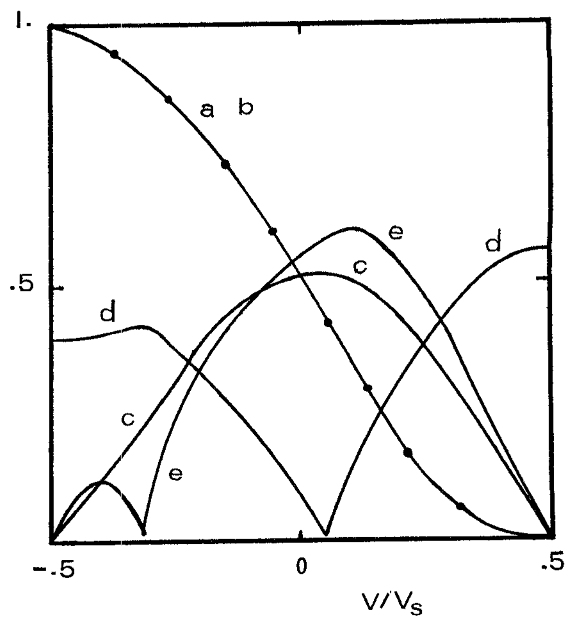

Fig. 3. The directional coupler modulator characteristic $f(V/V_s)$ of (6) and (8) (marked a) and the corresponding functions F_0, F_1, $|F_2|$, and F_3 of (27)–(30) (marked as b, c, d and e). —— (6); • (8).

that the proposed models accurately represent the characteristics of the directional coupler and the y-fed modulators of (5) and (6) respectively with an average root-mean-square error $< 1\%$

III. Harmonic and Intermodulation Analysis

Consider the case of a modulator driven by a multisinusoidal signal of the form

$$\frac{V}{V_s} = \frac{V_b}{V_s} + \sum_{k=1}^{K} \frac{V_k}{V_s} \sin w_k t. \tag{9}$$

If the modulator nonlinearity is characterized by

$$f\left(\frac{V}{V_s}\right) = A_0 + \sum_{n=1}^{6} A_n \cos\left(n\pi \frac{V}{V_s}\right)$$

then the normalized optical intensity output of the modulator can be expressed by

$$\frac{I}{I_0} = f\left(\frac{V}{V_s}\right) = A_0 + \sum_{n=1}^{6} A_n \cos\left[n\pi\left(\frac{V_b}{V_s} + \sum_{k=1}^{K} \frac{V_k}{V_s} \sin w_k t\right)\right]. \tag{10}$$

By repeated application of the trigonometric identities

$$\cos(a + b) = \cos a \cos b - \sin a \sin b$$

$$\sin(a + b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a \sin\psi) = 2 \sum_{n=0}^{\infty} J_{2n+1}(a) \sin(2n + 1)\psi$$

$$\cos(a \sin\psi) = J_0(a) + 2 \sum_{n=1}^{\infty} J_{2n}(a) \cos 2n\psi$$

TABLE I
COEFFICIENTS A_n, n = 0–6 OF THE TRUNCATED FOURIER-SERIES EXPANSIONS OF THE INTERFEROMETER, DIRECTIONAL COUPLER AND y-FED COUPLER MODULATORS OF (2), (5) AND (6), RESPECTIVELY

	A_0	A_1	A_2	A_3	A_4	A_5	A_6
Int	0.5	0.5	0.0	0.0	0.0	0.0	0.0
dc	0.4681	0.5024	0.0327	−0.0037	0.0001	0.0013	−0.0008
y-Fed	0.5210	0.5034	−0.0203	−0.0030	−0.0006	−0.0004	−0.0001

it can easily be shown that the amplitude of the normalized optical intensity output component of frequency

$$\sum_{k=1}^{K} \alpha_k w_k$$

will be given by

$$I_{\alpha_1,\alpha_2,\cdots,\alpha_K} = 2\sum_{n=1}^{6} A_n \cos\left(n\pi \frac{V_b}{V_s}\right) \prod_{k=1}^{K} J_{|\alpha_k|}\left(n\pi \frac{V_k}{V_s}\right) \quad (11)$$

for even-order products with $\Sigma_{k=1}^{K} |\alpha_k|$ = even integer and

$$I_{\alpha_1,\alpha_2,\cdots,\alpha_K} = 2\sum_{n=1}^{6} A_n \sin\left(n\pi \frac{V_b}{V_s}\right) \prod_{k=1}^{K} J_{|\alpha_k|}\left(n\pi \frac{V_k}{V_s}\right) \quad (12)$$

for odd-order products with $\Sigma_{k=1}^{K} |\alpha_k|$ = odd integer where $J_{|\alpha_k|}$ is the ordinary Bessel function of order $|\alpha_k|$. Also, it can be shown that the normalized output dc component will be given by

$$I_{dc} = A_0 + \sum_{n=1}^{6} A_n \cos\left(n\pi \frac{V_b}{V_s}\right) \prod_{k=1}^{K} J_0\left(n\pi \frac{V_k}{V_s}\right) \quad (13)$$

Using (11)–(13) the amplitudes of the output harmonics and intermodulation products of any even-order or odd-order can be calculated.

IV. SPECIAL CASE

To illustrate the use of (11)–(13), consider the special case of a two tone modulating signal of the form

$$V_m = V_0(\sin w_1 t + \sin w_2 t) = \frac{m_1 V_s}{2}(\sin w_1 t + \sin w_2 t) \quad (14)$$

Using (12), the amplitude of the normalized optical intensity output component of frequency w_i, i = 1, 2 will be

$$I_{1,0} = 2\sum_{n=1}^{6} A_n \sin\left(n\pi \frac{V_b}{V_s}\right) J_1\left(n\pi \frac{m_i}{2}\right) J_0\left(n\pi \frac{m_i}{2}\right) \quad (15)$$

and the amplitude of the normalized third-order intermodulation component of frequency $2w_1 \pm w_2$ (or $2w_2 \pm w_1$) will be

$$I_{2,1} = 2\sum_{n=1}^{6} A_n \sin\left(n\pi \frac{V_b}{V_s}\right) J_2\left(n\pi \frac{m_i}{2}\right) J_1\left(n\pi \frac{m_i}{2}\right), \quad (16)$$

and the amplitude of the normalized third-harmonic component of frequency $3w_i$ will be

$$I_{3,0} = 2\sum_{n=1}^{6} A_n \sin\left(n\pi \frac{V_b}{V_s}\right) J_3\left(n\pi \frac{m_i}{2}\right) J_0\left(n\pi \frac{m_i}{2}\right) \quad (17)$$

Using (11) the amplitude of the normalized second-order intermodulation component of frequency $w_1 \pm w_2$ will be

$$I_{1,1} = 2\sum_{n=1}^{6} A_n \cos\left(n\pi \frac{V_b}{V_s}\right)\left[J_1\left(n\pi \frac{m_i}{2}\right)\right]^2 \quad (18)$$

and the amplitude of the normalized second-harmonic component of frequency: $2w_i$ will be

$$I_{2,0} = 2\sum_{n=1}^{6} A_n \cos\left(n\pi \frac{V_b}{V_s}\right) J_2\left(n\pi \frac{m_i}{2}\right) J_0\left(n\pi \frac{m_i}{2}\right) \quad (19)$$

Also, from eqn (13) the normalized dc output component will be

$$I_{dc} = A_0 + \sum_{n=1}^{6} A_n \cos\left(n\pi \frac{V_b}{V_s}\right)\left[J_0\left(n\pi \frac{m_i}{2}\right)\right]^2 \quad (20)$$

Equations (15)–(20) are directly applicable for the interferometer and directional coupler modulators. For the y-fed coupler modulator V_b/V_s in (15)–(20) must be replaced by 0.5 + (V_b/V_s).

Using (15)–(19) the large signal intermodulation performance of the interferometer, directional coupler and y-fed coupler modulators can be calculated using the Bessel function built-in subroutines available in mainframe computers or using the Bessel function trigonometric approximations [5], [6], which are especially convenient for personal computers and pocket calculators users. Fig. 4 shows the results obtained using the data of Table I and (15)–(19) for a bias voltage (V_b/V_s) = 0.67 for the interferometer and directional coupler modulators and (V_b/V_s) = 0.04 for the y-fed coupler. The input voltage is 0 dB when m_i = 1. From Fig. 4 it is obvious that for small values of m_i the slope of the fundamental output is +1, the slope of the second-order intermodulation is +2 and the slope of the third-order intermodulation is +3. However, for large values of m_i the slopes deviate from the constant values.

For sufficiently small values of m_i, so that $3\pi m_i \ll 1$, the Bessel function $J_r(x)$ can be approximated by

$$J_r(x) = (x/2)^r/r!$$

and (15)–(20) reduce to

$$I_{1,0} = \frac{\pi}{2} m_i F_1 \quad (21)$$

$$I_{2,1} = \left(\frac{\pi}{4}\right)^3 m_i^3 F_3 \quad (22)$$

$$I_{3,0} = \frac{1}{12}\left(\frac{\pi}{4}\right)^3 m_i^3 F_3 \quad (23)$$

$$I_{1,1} = \frac{\pi^2}{8} m_i^2 F_2 \quad (24)$$

$$I_{2,0} = \frac{\pi^2}{32} m_i^2 F_2 \quad (25)$$

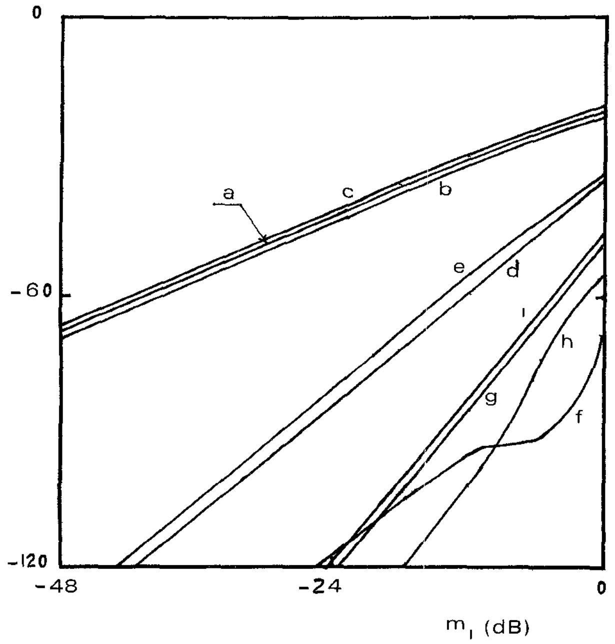

Fig. 4. Fundamental, second-order and third-order intermodulation performance of the interferometer (INT), the directional coupler (dc) and the y-fed modulators. 0 dB corresponds to $m_i = 1$. (a) Fundamental, INT. (b) Fundamental, dc. (c) Fundamental, y-fed. (d) Second-order intermodulation, INT. (e) Second-order intermodulation, dc. (f) Second-order intermodulation, y-fed. (g) Third-order intermodulation, INT. (h) Third-order intermodulation, dc. (i) Third-order intermodulation, y-fed.

and

$$I_{dc} = F_0 \tag{26}$$

where

$$F_0 = A_0 + \sum_{n=1}^{6} A_n \cos\left(n\pi \frac{V_b}{V_s}\right) \tag{27}$$

$$F_1 = \sum_{n=1}^{6} nA_n \sin\left(n\pi \frac{V_b}{V_s}\right) \tag{28}$$

$$F_2 = \sum_{n=1}^{6} n^2A_n \cos\left(n\pi \frac{V_b}{V_s}\right) \tag{29}$$

and

$$F_3 = \sum_{n=1}^{6} n^3A_n \sin\left(n\pi \frac{V_b}{V_s}\right). \tag{30}$$

Equations (27)–(30) are directly applicable for the interferometer and directional coupler modulators. For the y-fed coupler modulator, V_b/V_s (in (27)–(30) must be replaced by $0.5 + (V_b/V_s)$.

Using the data of Table I, (27)–(30) were evaluated, as functions of V_b/V_s, for the interferometer, directional coupler and the y-fed coupler modulators. The results are shown in Figs. 1–3. From Fig. 1 it is obvious that for $(V_b/V_s) = 0.5$, F_2 and consequently the second-order intermodulation product, and all the even-order products, are zero. At $(V_b/V_s) = 0.5$, the relative third-order intermodulation product $(I_{2,1}/I_{1,0}) = -73.96$ dB at $m_i = 0.02548$ (which corresponds to optical modulation depth $m_0 \simeq 0.04$). These results are in excellent agreement with the results obtained by Halemane *et al.* From Fig. 2 it is obvious that for $(V_0/V_s) = 0.45$, F_2 and consequently the second-order intermodulation products and all the even-order products are minumum (but not exactly zero). Also at $(V_b/V_s) = 0$, F_0 and F_2 and consequently the second-order intermodulation and the dc components are maximum. At the same bias point F_1 and F_3, and consequently the carrier and the third-order intermodulation components, are zero. At $(V_0/V_s) = 0.45$, the relative third-order intermodulation product $(I_{2,1}/I_{1,0}) = -72.05$ dB at $m_i = 0.02599$ (which corresponds to optical modulation depth $m_0 \simeq 0.04$). These results are in very close agreement with the results obtained by Halemane *et al.* From Fig. 3 it is obvious that for $(V_b/V_s) = 0.04$, F_2 and consequently the second-order intermodulation product and all the even-order products are minimum (but not exactly zero), and F_1 and F_3, and consequently the output carrier and third-order intermodulation components are maximum. At $(V_b/V_s) = 0.04$, the relative third-order intermodulation product $(I_{2,1}/I_{1,0})$ -73.53 dB at $m_0 \simeq 0.04$. These results are in very close agreement with the results obtained by Halemane *et al.*

V. Conclusion

In this paper empirical models have been presented for the nonlinear characteristics of interferometer, directional coupler and y-fed modulators. These models allow the prediction of the nonlinear performance of these modulators when driven by multisinusoidal large-amplitude input signals. The special case of relatively small-amplitude input signals was considered in detail and the results obtained in this paper are in excellent agreement with previously published results.

The results presented in this paper are directly applicable for the directional coupler and y-fed coupler modulators characterized by (5) and (6) respectively. Similar results can be obtained for the directional coupler and y-fed coupler modulators characterized by the general expressions of (3) and (4) respectively once the parameters of the corresponding truncated Fourier-series representation of (7) or (8) are obtained using the twelve-point method. It is worth mentioning here that the twelve-point method is a short-cut method for calculating the coefficients of the first seven terms of the Fourier-series representations of (7) and (8) using simple hand computations. The coefficients of Fourier-series representations using a larger or smaller number of terms can be obtained using standard curve-fitting subroutines built-in most mainframe computers; for example the IFLSQ of the IMSL library [7]. The use of seven-terms Fourier-series is a reasonable compromise between high accuracy and ease of computation.

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